Exponentials and Logarithms 1

The mass, A kg, of algae in a small pond is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.04t + 0.5$$

a. Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4 marks)

b. With reference to the model, interpret

i. the value of the constant p,

ii. the value of the constant q.

(2 marks)

c. Find, according to the model,

i. the mass of algae in the pond when $t = 4$, giving your answer to the nearest 0.5 kg

ii. the number of weeks it takes for the mass of algae in the pond to reach 7 kg.

(3 marks)

d. State one reason why this may not be a realistic model in the long term.

(1 mark)

a. Taking logs of both sides of the original equation gives:

$$\log_{10} A = \log_{10}(pq^t)$$
$$\log_{10} A = \log_{10} p + \log_{10} q^t$$
$$\log_{10} A = \log_{10} p + t\log_{10} q$$

and so:

$$\log_{10} p = 0.5 \text{ and } \log_{10} q = 0.04$$

1 mark

so:

$$p = 10^{0.5} = 3.162$$

1 mark

$$q = 10^{0.04} = 1.096$$

1 mark

leading to:

$$A = 3.162 \times 1.096^t$$

1 mark

bi. The constant p represents the initial mass of algae in the pond.

1 mark

bii. The constant q represents the ratio of algae from one week to the next.

1 mark

ci. Substitute $t = 4$ into the equation:

$$A = 3.162 \times 1.096^4 = 4.562... = 4.5 \text{ kg to the nearest } 0.5 \text{ kg}$$

1 mark

cii. Set $A = 7$ in the equation and solve for t:

$$7 = 3.162 \times 1.096^t$$

1 mark

$$1.096^t = \frac{7}{3.162}$$
$$\log_{10}(1.096^t) = \log_{10}\left(\frac{7}{3.162}\right)$$
$$t = \frac{\log_{10}\left(\frac{7}{3.162}\right)}{\log_{10} 1.096}$$
$$t = 8.669...$$
$$t = 8.7 \text{ weeks}$$

1 mark

d. The model predicts unlimited growth

1 mark