Trigonometry

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

i. Solve, for $0 < \theta \leq 450°$, the equation

$$4\cos^2\theta = 5\sin\theta$$

giving your answers to one decimal place.

(5 marks)

ii. a. A student's attempt to solve the question

"Solve, for $-90° < x < 90°$, the equation $2\tan x - 5\sin x = 0$"

Is set out below.

$$2\tan x - 5\sin x = 0$$
$$2\frac{\sin x}{\cos x} - 5\sin x = 0$$
$$2\sin x - 5\sin x\cos x = 0$$
$$2 - 5\cos x = 0$$
$$\cos x = \frac{2}{5}$$
$$x = 66.4°$$

Identify two errors or omissions made by the student, giving a brief explanation of each.

(2 marks)

The first four positive solutions, in order of size, of the equation

$$\cos(6\alpha + 20°) = \frac{2}{5}$$

are α_1, α_2, α_3 and α_4

b. Find, to the nearest degree, the value of α_4

(2 marks)

i. Begin by writing the equation in terms of just sin.
Use $\cos^2\theta = 1 - \sin^2\theta$ for this.

1 mark

$4\cos^2\theta = 5\sin\theta \Rightarrow 4(1 - \sin^2\theta) = 5\sin\theta \Rightarrow 4\sin^2\theta + 5\sin\theta - 4 = 0$

1 mark

Solve the resulting quadratic to give:

$\sin\theta = \frac{-5+\sqrt{89}}{8}$ – can ignore $\frac{-5-\sqrt{89}}{8}$ as it is < -1

1 mark

The calculator value of θ is 33.7°

1 mark

The other values in the required region are 146.7° (180 – 33.7) and 393.7° (360 + 33.7).

1 mark

iia. Cancelling by sin x will result in missing solutions

1 mark

Not all solutions of $\cos x = \frac{2}{5}$ have been found. $x = -66.4°$ is missing

1 mark

iib. The first four positive solutions come from the 66.4°, 360° – 66.4°, 360° + 66.4° and 720° – 66.4°.
Set $6\alpha + 20° = 720° - 66.4°$

1 mark

$$6\alpha + 20° = 653.6°$$
$$6\alpha = 633.6°$$
$$\alpha = 105.6°$$

So $\alpha = 106°$ to the nearest degree.

1 mark