Vectors

Relative to a fixed origin *O*,

- point *P* has position vector 6**i** – 2**j**
- point *Q* has position vector 4**i** – **j**

a. Find $\overrightarrow{PQ}$

(2 marks)

Given that *R* is the point such that $\overrightarrow{QR}$ = 4**i** + 8**j**

b. Show that angle *PQR* = 90°

(2 marks)

Given also that *S* is the point such that $\overrightarrow{PS} = 3\overrightarrow{QR}$

c. find the exact area of *PQRS*

(4 marks)

a.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$$
$$= \overrightarrow{OQ} - \overrightarrow{OP}$$
$$= (4i - j) - (6i - 2j)$$

1 mark

$$= -2i + j$$

1 mark

b.

The gradient of $PQ = -\frac{1}{2}$ and the gradient of $QR = \frac{8}{4} = 2$

1 mark

$-\frac{1}{2} \times 2 = -1$ so PQ and QR are perpendicular to each other ⇒ angle $PQR = 90°$

1 mark

c. *PQRS* is a trapezium with *PS* and *QR* being the parallel sides and *PQ* the 'height'.

$$\left|\overrightarrow{PQ}\right| = \sqrt{(-2)^2 + 1^2}$$
$$= \sqrt{5}$$

1 mark

$$\left|\overrightarrow{QR}\right| = \sqrt{4^2 + 8^2}$$
$$= 4\sqrt{5}$$
$$\left|\overrightarrow{PS}\right| = 12\sqrt{5}$$

1 mark

$$\text{Area} = \frac{1}{2} \times \left(4\sqrt{5} + 12\sqrt{5}\right) \times \sqrt{5}$$

1 mark

$$\text{Area} = 40$$

1 mark