Factor Theorem

$$f(x) = 3x^3 + 9x^2 + 17x + 11$$

a. Use the factor theorem to show that (x + 1) is a factor of f(x).

(2 marks)

b. Find the constants *a*, *b* and *c* such that

$$f(x) = (x + 1)(ax^2 + bx + c)$$

(2 marks)

c. Hence show that f(x) = 0 has only one real root.

(2 marks)

d. Write down the real root of the equation f(x + 2) = 0

(1 mark)

a. Use the factor theorem to show that f(-1) = 0:

$$f(-1) = 3(-1)^3 + 9(-1)^2 + 17(-1) + 11$$
$$= -3 + 9 - 17 + 11$$
$$= 0$$

1 mark

As f(-1) = 0, $(x + 1)$ is a factor of $f(x)$

1 mark

b. Use algebraic long division

$$\begin{array}{r} 3x^2 + 6x + 11 \\ x + 1 \overline{)3x^3 + 9x^2 + 17x + 11} \\ 3x^3 + 3x^2 \qquad\qquad\quad \\ 6x^2 + 17x \qquad\quad \\ 6x^2 + 6x \qquad\quad \\ 11x + 11 \\ 11x + 11 \\ 0 \end{array}$$

$$f(x) = (x + 1)(3x^2 + 6x + 11)$$

1 mark for any 2 of 3, 6 or 11, 2 marks for all 3

c. Use the discriminant to show that there are no real roots for the quadratic factor:

$$b^2 - 4ac = 6^2 - 4 \times 3 \times 11$$

1 mark

$$36 - 132 = -96 < 0 \Rightarrow \text{no real roots}$$

1 mark

d. Use transformations to find the real root of the equation $f(x + 2) = 0$.

$y = f(x + 2)$ is the function $y = f(x)$ translated by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

As the real root of the equation f(x) = 0 is $x = -1$, the real root of the equation $f(x + 2) = 0$ is $x = -3$

1 mark