Area Under a Curve

A curve C has equation $y = f(x)$ where

$$f(x) = -2x^2 + 8x + 3$$

a. Write $f(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3 marks)

The curve C has a maximum turning point at M.

b. Find the coordinates of M.

(2 marks)

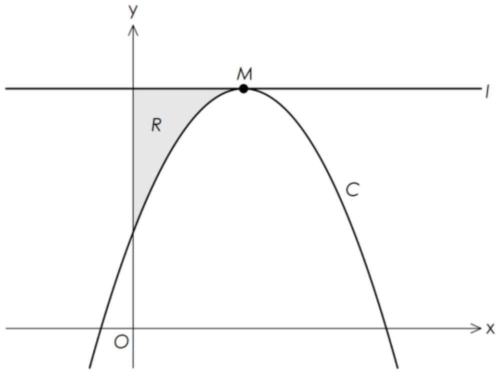

Figure 1

Figure 1 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 1, is bounded by C, l and the y-axis.

c. Using algebraic integration, find the area of R.

(5 marks)

a. $-2x^2 + 8x + 3 = -2[x^2 - 4x] + 3$ 1 mark

$= -2[(x - 2)^2 - 4] + 3$ 1 mark

$= -2(x - 2)^2 + 11$ 1 mark

b. The maximum value of y occurs when $(x - 2)^2 = 0$, which is when $x = 2$ 1 mark

When $(x - 2)^2 = 0$, $y = 11$, so the coordinates of $M = (2, 11)$ 1 mark

c. To find the area of R, calculate the area bound by the x-axis, the y-axis, the line l ($y = 11$) and the line $x = 2$, and subtract the area under the curve C, bound by the x-axis, the y-axis and the line $x = 2$.

For the area under the curve C, integrate:

$$\int_0^2 -2x^2 + 8x + 3 \, dx$$

1 mark

$$= \left[-\frac{2}{3}x^3 + 4x^2 + 3x\right]_0^2$$

1 mark

$$R = 2 \times 11 - \int_0^2 -2x^2 + 8x + 3 \, dx$$

1 mark

$$R = 22 - \frac{50}{3}$$

1 mark

$$R = \frac{16}{3}$$

1 mark