Triangles

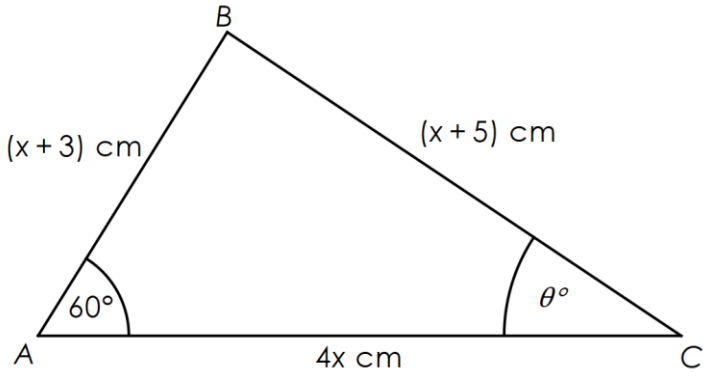

Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 3)$ cm, $BC = (x + 5)$ cm, $AC = 4x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

a. i. Show that $3x^2 - 4x - 4 = 0$

(3 marks)

ii. Hence find the value of x.

(1 mark)

b. Hence find the value of θ giving your answer to one decimal place.

(2 marks)

ai. Use the Cosine Rule to form an equation:

$$(x + 5)^2 = (x + 3)^2 + (4x)^2 - 2(x + 3)(4x)\cos 60°$$

1 mark

Expand and simplify to give:

$$x^2 + 10x + 25 = x^2 + 6x + 9 + 16x^2 - 2(4x^2 + 12x) \times \frac{1}{2}$$

$$x^2 + 10x + 25 = x^2 + 6x + 9 + 16x^2 - 4x^2 - 12x$$

$$0 = 12x^2 - 16x - 16$$

1 mark

Leading to:

$$3x^2 - 4x - 4 = 0$$

1 mark

aii. Factorise the quadratic to give:

$$3x^2 - 4x - 4 = (3x + 2)(x - 2)$$

Leading to $x = -\frac{2}{3}$ or $x = 2 \Rightarrow x = 2$ as $x > 0$

1 mark

b. The sides of the triangle can now be written as $AB = 5$ cm, $BC = 7$ cm and $AC = 8$ cm. Using the Sine Rule gives:

$$\frac{\sin\theta}{5} = \frac{\sin 60°}{7} \Rightarrow \sin\theta = \frac{5\sin 60°}{7}$$

1 mark

$$\theta = 38.2°$$

1 mark