

Integration

Find the value of the constant k , $0 < k < 16$, such that

$$\int_k^{16} \frac{8}{\sqrt{x}} dx = 40$$

(4 marks)

Rewrite the integral using index notation:

$$\int_k^{16} \frac{8}{\sqrt{x}} dx = \int_k^{16} 8x^{-\frac{1}{2}} dx$$

Integrate using $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$\int_k^{16} 8x^{-\frac{1}{2}} dx = \left[16x^{\frac{1}{2}} \right]_k^{16}$$

1 mark

$$16 \times 16^{\frac{1}{2}} - 16 \times k^{\frac{1}{2}} = 64 - 16\sqrt{k}$$

$$64 - 16\sqrt{k} = 40$$

1 mark

$$16\sqrt{k} = 24$$

$$\sqrt{k} = \frac{24}{16}$$

$$\sqrt{k} = \frac{3}{2}$$

1 mark

$$k = \frac{9}{4}$$

1 mark