Binomial Expansion

The binomial expansion of

$$(1 + ax)^{12}$$

up to and including the term in x^2 is

$$1 - \frac{48}{7}x + kx^2$$

where a and k are constants.

a. Show that $a = -\frac{4}{7}$

(2 marks)

b. Hence find the value of k

(2 marks)

Using the expansion and making your method clear,

c. find an estimate for the value of $\left(\frac{15}{14}\right)^{12}$, giving your answer to 4 decimal places.

(2 marks)

a. Using the binomial expansion, the coefficient of the x term is given by:

$$^{12}C_1 \times a = -\frac{48}{7}$$

1 mark

$$12 \times a = -\frac{48}{7} \Rightarrow a = -\frac{4}{7}$$

1 mark

b. Again, using the binomial expansion, the coefficient of the x^2 is given by:

$$^{12}C_2 \times a^2 = k \Rightarrow 66 \times \frac{16}{49} = k$$

1 mark

$$k = \frac{1056}{49}$$

1 mark

c. Find the value of x by solving:

$$1 - \frac{4}{7}x = \frac{15}{14} \Rightarrow x = -\frac{1}{8}$$

Substitute into the binomial expansion:

$$\left(\frac{15}{14}\right)^{12} = 1 - \frac{48}{7} \times \left(-\frac{1}{8}\right) + \frac{1056}{49} \times \left(-\frac{1}{8}\right)^2$$

1 mark

which gives:

$$\left(\frac{15}{14}\right)^{12} = 2.1939 \text{ to 4 decimal places}$$

1 mark