Straight Lines

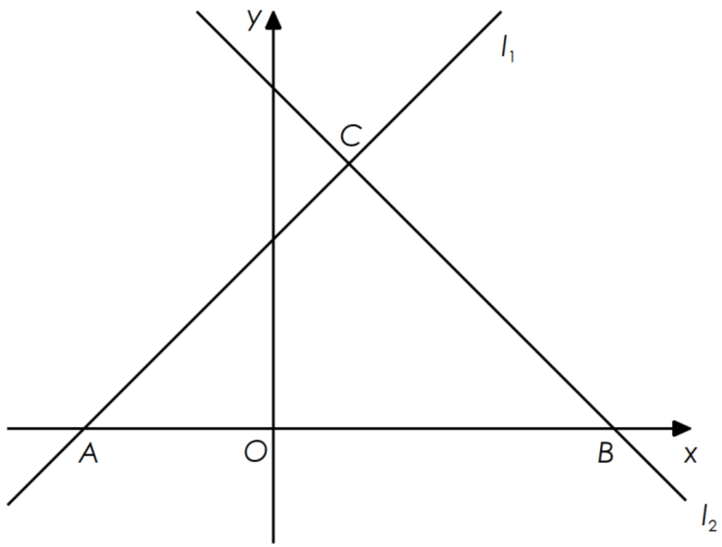

Figure 1

The line l_1 has equation $y = \frac{5}{6}x + 15$

The line l_2 is perpendicular to l_1 and passes through the point $B(15, 0)$, as shown in Figure 1.

a. Show that an equation for the line l_2 is

$$6x + 5y = 90$$

(3 marks)

Given that

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x-axis at the point A

b. find the exact area of triangle ABC, giving your answer as a fully simplified fraction in the form $\frac{p}{q}$

(5 marks)

a. The two lines are perpendicular to each other, therefore the products of their gradients will be -1.

So the gradient of l_2 is $-\frac{6}{5}$

1 mark

As l_2 passes through the point (15, 0):

$$y - 0 = -\frac{6}{5}(x - 15)$$

1 mark

$$5y = -6x + 90$$
$$6x + 5y = 90$$

1 mark

b. l_1 crosses the x-axis when $y = 0$:

$$\frac{5}{6}x + 15 = 0$$

$$x = -18$$

Coordinates of $A = (-18, 0)$

1 mark

Solving the equations for l_1 and l_2 simultaneously for the value of y:

1 mark

$$6y - 5x = 90$$
$$6x + 5y = 90$$
$$y = \frac{990}{61}$$

1 mark

$$\text{Area of triangle} = \frac{1}{2} \times (15 + 18) \times \frac{990}{61}$$

1 mark

$$= \frac{16335}{61}$$

1 mark