Exponentials and Logarithms 2

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.8e^{-0.4t} \qquad t \in \mathbb{R} \quad t \geq 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.6 kg/cm^2

a. state the value of k.

(1 mark)

From the instant when the tyre developed the puncture,

b. find the time taken for the air pressure to fall to 1 kg/cm^2
Give your answer in minutes to one decimal place.

(3 marks)

c. Find the rate at which the air pressure in the tyre is decreasing exactly 3 minutes from the instant when the tyre developed the puncture.
Give your answer in kg/cm^2 per minute to 3 significant figures.

(2 marks)

a. k can be found when $t = 0$ and $P = 2.6$

$$2.6 = k + 1.8$$
$$k = 0.8$$

1 mark

b. Set $P = 1$ in the equation and solve for t:

$$1 = 0.8 + 1.8e^{-0.4t}$$
$$1.8e^{-0.4t} = 0.2$$

1 mark

$$e^{-0.4t} = \frac{0.2}{1.8}$$
$$-0.4t = \ln\left(\frac{0.2}{1.8}\right)$$
$$t = \frac{\ln\left(\frac{0.2}{1.8}\right)}{-0.4}$$

1 mark

$t = 5.5$ minutes

1 mark

c. The rate at which the tyre pressure changes is found by differentiating the equation for P with respect to t.

$$\frac{dP}{dt} = -0.72e^{-0.4t}$$

Substitute $t = 3$ to find the rate the tyre is decreasing at 3 minutes.

$$\frac{dP}{dt} = -0.72e^{-0.4 \times 3}$$

1 mark

The tyre is decreasing at a rate of 0.217 kg/cm^2 per minute.

1 mark