Proof

i. A student states

"if x^2 is greater than 1 then x must be greater than 1"

Determine whether or not this statement is true, giving a reason for your answer.

(1 mark)

ii. Prove that for all positive integers n,

$$n^3 - 5n^2 + 6n$$

is always even.

(3 marks)

i. Pick a negative value of $x < -1$

$x = -2 \Rightarrow x^2 = (-2)^2 = 4 > 1$ but $-2 < 1$ so the statement is not true.

1 mark

ii. Begin by factorising the expression:

$$n^3 - 5n^2 + 6n = n(n^2 - 5n + 6)$$
$$= n(n - 2)(n - 3)$$

1 mark

$n - 2$ and $n - 3$ are consecutive integers, so one of them will be a multiple of 2

1 mark

So, $n(n - 2)(n - 3)$ is a product that contains one or two even numbers, and any product containing an even number will result in an even number.

1 mark