

Circles

The circle C has equation

$$x^2 + y^2 + 18x + 4y + k = 0$$

where k is a constant.

a. Find the coordinates of the centre of C.

(2 marks)

Given that C does not cut or touch the x-axis,

b. find the range of possible values for k .

(3 marks)

a. To find the coordinates of the centre of C, rearrange the equation for C into the form:

$$(x + a)^2 + (y + b)^2 = r^2$$

Do this by completing the square:

$$\begin{aligned}x^2 + y^2 + 18x + 4y + k &= x^2 + 18x + y^2 + 4y + k \\&= (x + 9)^2 + (y + 2)^2 - 9^2 - 2^2 + k \\&= (x + 9)^2 + (y + 2)^2 - 85 + k\end{aligned}$$

1 mark

Which gives the coordinates of the centre of C as:

$$(-9, -2)$$

1 mark

b. As C does not cut or touch the x-axis, the radius of the circle must be less than 4 (the y-coordinate of the centre of the circle). So:

$$k > 81$$

1 mark

The radius of the centre of the circle also has to be greater than 0, so:

$$\begin{aligned}85 - k &> 0 \\k &< 85\end{aligned}$$

1 mark

$$81 < k < 85$$

1 mark